B.A./B.Sc. 5th Semester (Honours) Examination, 2019 (CBCS)

Subject : Mathematics

Paper : BMH5CC11

(Partial Differential Equations and Applications)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

[Symbols and notation have their usual meaning.]

1. Answer *any ten* questions from the following: 2×10=20

(a) Define a first order quasi linear equation and provide an example of it.

(b) Obtain the solution of $zp + x = 0$.

(c) Use the separation of variables $u(x, y) = f(x) + g(y)$ to solve $u_x^2 + u_y^2 = 1$.

(d) Define Cauchy problem for second order partial differential equation with example.

(e) Find the differential equation of all spheres of radius r having centre in the xy plane.

(f) Obtain the solution of $u_x - u_y = 1$ with $u(x, 0) = x^2$.

(g) Obtain the partial differential equation which has its general solution $u = f\left(\sqrt{x^2 + y^2}\right)$, f being an arbitrary function.

(h) Determine the region where the given PDE: $x\,u_{xx} + u_{yy} = x^2$ is hyperbolic, parabolic or elliptic.

(i) Prove that the characteristic curves for the equation $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = u$ in $x - y$ plane is circles with centre at origin.

(j) Show that $u(x, t) = x - e^{-t}$ is a bounded solution of $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + e^{-t}$ in $0 \le t < \infty$.

(k) Eliminate the arbitrary functions f and g from $y = f(x - at) + g(x + at)$.

(l) Find the nature of the partial differential equation $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right) z = 0$.

(m) State Cauchy-Kowalevsky theorem.

(n) Verify that $U(x, t) = 1 - e^{-t}\left(1 - f(xe^t)\right)$ with $U(x, 0) = f(x)$ is the solution of $\frac{\partial U}{\partial t} = x\frac{\partial U}{\partial x} - 1 + U$.

(o) Find the characteristics of the equation: $u_{xx} + 2u_{xy} + \sin^2 x\, u_{yy} + u_y = 0$ .When is it of hyperbolic type?

2. Answer *any four* questions from the following: 5×4=20

(a) Find the integral surface of the linear partial differential equation $x(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ containing the straight line $x+y=0, z=1$ where $p=\frac{\partial z}{\partial x}; q=\frac{\partial z}{\partial y}$

(b) Obtain the general solution of heat flow equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$ by the method of separation of variables.

(c) Show that the equation $x^2 z_{xx} - y^2 z_{yy} = 0$ is of hyperbolic type. Find its characteristics. 2+3=5

(d) (i) Solve $z(x+y)p + z(x-y)q = x^2+y^2$.

(ii) Give geometrical interpretation of $P\,p + Q\,q = R$. 3+2=5

(e) Reduce the equation $3\,u_{xx} + 10\,u_{xy} + 3\,u_{yy} = 0$ to its canonical form and hence solve it. 2+3=5

(f) Solve the problem by the method of characteristic

$$pz + q = 1 \text{ with initial data } y = x, z = \frac{x}{2}.$$

3. Answer *any two* questions from the following: 10×2=20

(a) (i) Obtain the integral surface of the equation

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0 \text{ satisfying the condition } u(1,y) = y.$$

(ii) Solve $z\,(x+y)\,p + z\,(x-y)\,q = x^2+y^2$. 5+5=10

(b) (i) Solve $u_{tt} = c^2\,u_{xx}, x>0, t>0$ subject to the non-homogeneous boundary conditions
$u(x,0) = f(x), x \geq 0,$
$u_t(x,0) = g(x), x \geq 0,$
$u(0,t) = p(t), t \geq 0.$

(ii) Solve $u_{tt} - c^2\,u_{xx} = 0\;(-\infty < x < \infty, t \geq 0)$ subject to the initial conditions $u(x,0) = \eta(x)$ and $u_t(x,0) = v(x)$. 6+4=10

(c) (i) Reduce $x^2\,z_{xx} + 2xy\,z_{xy} + y^2\,z_{yy} = 0$ to canonical form and hence solve it.

(ii) Find the characteristic of $x^2\,z_{xx} - y^2\,z_{yy} = 0$. (5+3)+2=10

(d) (i) Solve by method of separation of variables $\frac{\partial U}{\partial x} = 2\frac{\partial U}{\partial t} + U$

$$U(x,0) = 6e^{-3x}$$

(ii) Prove that the surface passing through the parabola $U=0, y^2=4ax$ and $U=1, y^2=-4ax$ and satisfying the equation $x\frac{\partial^2 U}{\partial x^2} + 2\frac{\partial U}{\partial x} = 0$ 5+5=10

$$\text{is } U = -\frac{y^2}{8ax} + \frac{1}{2}$$
