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3 Yr. Degree/4 Yr. Honours 1st Semester Examination, 2023 (CCFUP)

Subject : Mathematics

Course : MATH1021 (MINOR)

(Calculus, Geometry & Vector Calculus)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols have their usual meaning.

1. Answer *any ten* of the following questions: 2×10=20

(a) Find the surface area of a sphere generated by revolution of a circle $x^2+y^2=a^2$ about x axis.

(b) Evaluate $\lim_{x\to 0} \frac{x\cos x-\log(1+x)}{x^2}$.

(c) Find the asymptotes parallel to axes of the curve $(x^2+y^2)x-ay^2=0$.

(d) Find the envelope of $y^2=m^2(x-m)$, m being the parameter.

(e) Obtain the reduction formula for $\int x^n e^{ax}\,dx$, n being a positive integer.

(f) Evaluate $\int_0^1 x^4\sqrt{1-x^2}\,dx$.

(g) Evaluate $\int_0^{\frac{\pi}{2}} \cos^5 x\,dx$.

(h) Find the points on $\frac{8}{r}=3-\sqrt{2}\cos\theta$ whose radius vector is 4.

(i) Find the nature of the conic $\frac{5}{r}=2-2\cos\theta$ and find its semilatus rectum.

(j) Determine the centre of the conic $3x^2+4y^2-12x+8y+4=0$.

(k) Find the centre and the radius of the sphere $3x^2+3y^2+3z^2+2x-4y-2z-1=0$.

(l) Show that $[\vec{i}-\vec{j},\vec{j}-\vec{k},\vec{k}-\vec{i}]=0$.

(m) If $\vec{r}=\sin t\,\vec{i}+\cos t\,\vec{j}+2\vec{k}$, then show that $\left|\frac{d^2\vec{r}}{dt^2}\right|=1$.

(n) Examine whether the vectors $7\vec{i}-9\vec{j}+11\vec{k}, 3\vec{i}+\vec{j}-5\vec{k}, 5\vec{i}-21\vec{j}+37\vec{k}$ are coplanar.

(o) Show that $\nabla^2\left(\frac{1}{r}\right)=0$, where $r=\sqrt{x^2+y^2+z^2}$.

2. Answer *any four* of the following questions: 5×4=20

(a) If $y=e^{m\sin^{-1}x}$, establish $(1-x^2)y_{n+2}-(2n+1)x\,y_{n+1}-(n^2+m^2)y_n=0$, symbols used are usual meaning.

(b) Determine $\lim_{x\to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$.

(c) Show that the arc of the upper half of the cardioid $r = a\,(1-\cos\theta)$ is bisected at $\theta = \frac{2\pi}{3}$. Show also that the perimeter of the curve is $8a$. 4+1

(d) Let PSP' be a focal chord of a conic $\frac{l}{r} = 1 - e\cos\theta$. Prove that the angle between the tangents at P and P' is $\tan^{-1}\frac{2e\sin\alpha}{1-e^2}$, where α is the angle between the chord and the major axis.

(e) Find the equation of the cylinder, whose generators are parallel to $\frac{x}{-1}=\frac{y}{2}=\frac{z}{3}$ and whose guiding curve is $x^2+y^2=9, z=1$.

(f) Find div $\vec{F}$ and curl $\vec{F}$ where $\vec{F} = \text{grad}\,(x^3+y^3+z^3-3xyz)$.

3. Answer *any two* of the following questions: 10×2=20

(a) (i) Find the asymptotes of $y^3-6xy^2+11x^2y-6x^3+x+y=0$.

(ii) Find the values of a and b in order that $\lim_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{x^3}$ may equal to 1. 6+4

(b) (i) If $I_n = \int \tan^n x\,dx$, where $n\in\mathbb{N}-\{1\}$, then show that $I_n = \frac{\tan^{n-1}x}{n-1} - I_{n-2}$. Hence find $\int \tan^4 x\,dx$.

(ii) Find the length of one arc of the cycloid $x = a\,(\theta-\sin\theta), y = a\,(1-\cos\theta)$. (4+2)+4

(c) (i) Reduce the equation $3x^2+2xy+3y^2-16x+20=0$ in canonical form and then determine its nature.

(ii) Find polar equation of the tangent to the conic $\frac{2}{r} = 1-\cos\theta$ at $\theta = \frac{1}{2}\pi$. 6+4

(d) (i) If $\vec{r} = 3t\,\vec{i} + 3t^2\vec{j} + 2t^3\vec{k}$, then find the value of $\left[\frac{d\vec{r}}{dt}, \frac{d^2\vec{r}}{dt^2}, \frac{d^3\vec{r}}{dt^3}\right]$.

(ii) If $\vec{f}$ be a continuously differentiable vector valued function, then show that div curl $\vec{f} = 0$.

(iii) If $\varphi(x,y,z) = x^2yz + 4xz^2$, then find grad φ at $(1,-2,-1)$.

(iv) If $\vec{r} = \sin t\,\vec{i} - \cos t\,\vec{j} - \vec{k}$ and $\vec{s} = \cos t\,\vec{i} + \sin t\,\vec{j} + t\,\vec{k}$, find $\frac{d}{dt}(\vec{r}\times\vec{s})$. 3+3+2+2

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